In a short form we can write

$$Z_i = (1 + j\alpha k)Z_r, \tag{68}$$

where

$$\alpha = 4\pi r - 2 + 3c\pi p - \frac{2c\pi}{p}. \tag{69}$$

According to these equations perfect match $Z_i = Z_r$ can be obtained at the operating frequency ($k=0$), as it was anticipated. The reactive component is determined by α which is a function of the impedance ratio p.

In order to set up the relation between the VSWR, R and α, we first calculate the reflection coefficient

$$\Gamma = \frac{Z_i - Z_r}{Z_i + Z_r}. \tag{70}$$

If we use the relation of (68) we obtain

$$\Gamma = \frac{j\alpha k}{2 + j\alpha k}. \tag{71}$$

The VSWR can now be expressed in terms of Γ[1]

$$R = \frac{1 + |\Gamma|}{1 - |\Gamma|}. \tag{72}$$

Substituting (71) in (72) gives

$$R = \frac{\sqrt{4 + \alpha^2 k^2} + \alpha k}{\sqrt{4 + \alpha^2 k^2} - \alpha k}. \tag{73}$$

Neglecting higher orders of k than the first we obtain the simple function

$$R = 1 + \alpha k. \tag{74}$$

Therefore, α directly determines the slope of the vswr curve around the operating frequency.

[1] C. G. Montgomery, "Technique of Microwave Measurements," Radiation Laboratory Series, vol. 11, McGraw-Hill Book Co., Inc., New York, N. Y.; 1947.

Transverse Electric Resonances in a Coaxial Line Containing Two Cylinders of Different Dielectric Constant

J. W. CARR†

Summary—A coaxial line containing a medium of propagation consisting of two coaxial cylindrical cylinders of different dielectric constant is considered for the special case of TE_{nm} resonances, and numerical calculations are carried out for a few cases of the TE_{11} type resonance. A reference paper called to the attention of the author by the reviewer treats the general condition of propagation in such a line. The numerical solutions to the cases of interest in this application were not performed, however, presumably since the interest was in propagating modes and since the general characteristic equation is quite complicated. It is shown here that consideration of transverse boundary conditions only leads to an equation which is much less complicated and which is equivalent to the reduction of the general characteristic equation (of the reference paper) when cutoff is approached.

Introduction

Using coaxial chokes in the upper microwave region, difficulty is encountered due to the presence of higher-order mode resonances in the choke sections. At these shorter wavelengths it is quite difficult to maintain a rugged mechanical construction of the transmission system, including the choke, and prevent occurrence of the circumferential TE_{11} resonance in the choke sections while operating the system over a broad band. In design literature ways of damping out this undesirable resonance are indicated. However, this usually leads to a tedious experimental project and results in a choke design which is quite complicated and not very rugged mechanically. For the ratios of outer to inner diameters used in such chokes containing a single homogeneous isotropic medium the TE_{11} and TE_{21} resonances are the first two that are excited in the choke as the operating frequency is increased. They are separated spectrally by a nearly two-to-one ratio for the diameter ratios (under 2 to 1) used in chokes.

If we are justified in assuming that a medium consisting of two concentric cylinders of different dielectric constant maintains essentially the same relative spectral separation between the TE_{11} and TE_{21} resonances, then only the calculations for the TE_{11} resonant conditions need be made.

Rather than make the whole structure very small, or use the resonance damping technique, the effective medium of the choke can be changed by partial dielectric loading (partial to allow clearance for mechanical rotation and eccentricities) so that the TE_{11} resonance comes in below the band of operation and the TE_{21} above the band. The resulting design can be kept quite

† Gilfillan Brothers, Inc., Los Angeles, Calif.

simple mechanically and operation over a large bandwidth free of interfering asymmetrical resonances can be obtained.

Considerations of the Problem

Let us write the wave equation

$$\nabla \times \nabla \times \bar{E} = \beta^2 \bar{E}. \tag{1}$$

Let $\bar{E} = \nabla \times \bar{\pi}$, as it must to satisfy (1). Then for fields of transverse E-components only we can set $\bar{\pi} = \pi_z$ and solve for π_z from which, through the relations of Maxwell's equations, all the other necessary components for TE waves can be obtained. Setting $\bar{\pi} = \pi_z$ demands a solution (if a solution exists) with only r and ϕ components in a coaxial line as in Fig. 1. Also this demands, for a coaxial line, an axial component H_z. That is, the TEM dominant mode relations are not obtained by using the electric Hertz vector $\bar{E} = \nabla \times \pi_z$ but rather from the magnetic Hertz vector $\bar{H} = \nabla \times \pi_z$ (for a single homogeneous isotropic medium).

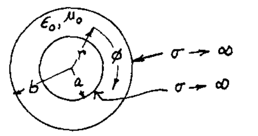

Fig. 1

For such a single medium coaxial line we obtain.

$$\pi_z = [FJ_n(r\sqrt{\beta^2 - h^2}) + GN_n(r\sqrt{\beta^2 - h^2})]C \cos n\phi A e^{-jhz}, \tag{2}$$

and, applying Maxwell's equations to this co-ordinate system,

$$E_r = \frac{1}{r}\frac{\partial \pi_z}{\partial \phi} = \frac{-n}{r}[FJ_n(r\sqrt{\beta^2 - h^2}) + GN_n(r\sqrt{\beta^2 - h^2})]C \sin n\phi A e^{-jhz}, \tag{3a}$$

$$E_\phi = \frac{-\partial \pi_z}{\partial r} = -\sqrt{\beta^2 - h^2}[FJ_n'(r\sqrt{\beta^2 - h^2}) + GN_n'(r\sqrt{\beta^2 - h^2})]C \cos n\phi A e^{-jhz}, \tag{3b}$$

where $\beta = 2\pi/\lambda_0$ = free space propagation constant and h = propagation constant in axial (z) direction along the line.

Now consider a coaxial line with two different concentric cylinders of different dielectric constant ϵ_1 and ϵ_2, with $\mu_1 = \mu_2 = \mu_0$.

If only the TE_{nm} mode resonances (cutoffs) are of interest then it is necessary to apply only the transverse boundary conditions.

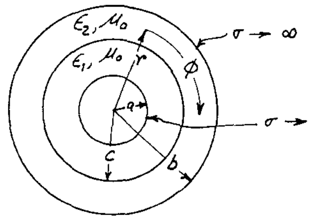

Fig. 2

Applying (3a) and (3b) with the respective subscripts to mediums 1 and 2 in Fig. 2 and the following boundary conditions for all ϕ and z:

$$E_{\phi 1} = 0 \text{ at } r = a$$

$$E_{\phi 2} = 0 \text{ at } r = b$$

$$Dr_1 = \epsilon_1 Er_2 \Big|_{r=c} = \epsilon_2 Er_2 \Big|_{r=c} = Dr_2$$

$$E\phi_1 \Big|_{r=c} = E\phi_2 \Big|_{r=c}.$$

The following equation is obtained for $n = 1$:

$$\frac{N_1(ck_{1c})J_1'(ak_{1c}) - N_1'(ak_{1c})J_1(ck_{1c})}{N_1\left(ck_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)J_1'\left(bk_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) - N_1'\left(bk_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)J_1\left(ck_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\,\frac{N_1'(ck_{1c})J_1'(ak_{1c}) - N_1'(ak_{1c})J_1'(ck_{1c})}{N_1'\left(ck_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)J_1'\left(bk_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) - N_1'\left(bk_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)J_1'\left(ck_{1c}\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)}, \tag{4}$$

where we have set $h_1 = h_2 = 0$, since we are considering the cutoff condition where $h \to 0$.

$$k_{1c} = \frac{2\pi}{\lambda_{1c}}$$

$$k_{2c} = \frac{2\pi}{\lambda_{2c}}.$$

Setting $n = 1$ and taking the first root of (4) gives the conditions for TE_{11} resonance.

The following families of curves give solutions for the cases noted for TE_{11} resonance. The resonant frequency f_c for the TE_{11} condition can be readily obtained by obtaining λ_c from the curves. This gives design values for a/b and c/b for the particular values of ϵ_2/ϵ_1 for which the calculations have been made.

$$F_c = \frac{c}{\lambda_c}$$

$$\epsilon' = \frac{\epsilon}{\epsilon_0}.$$

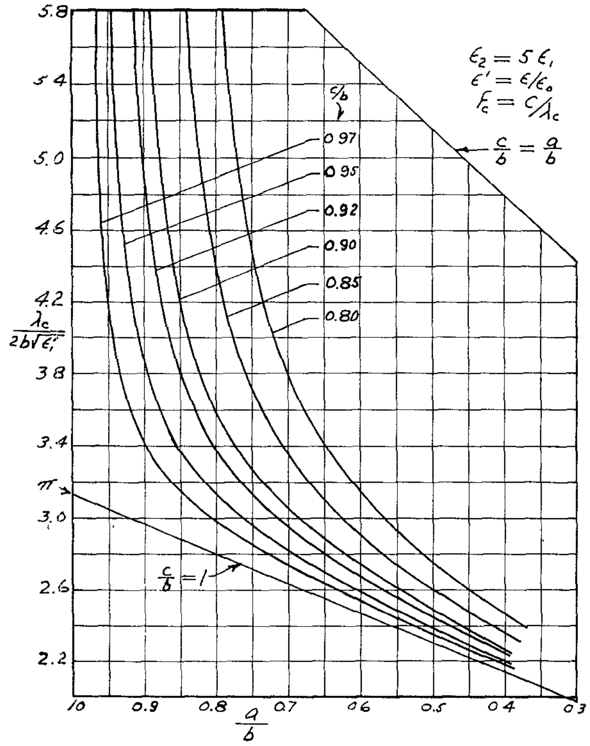

Fig. 3

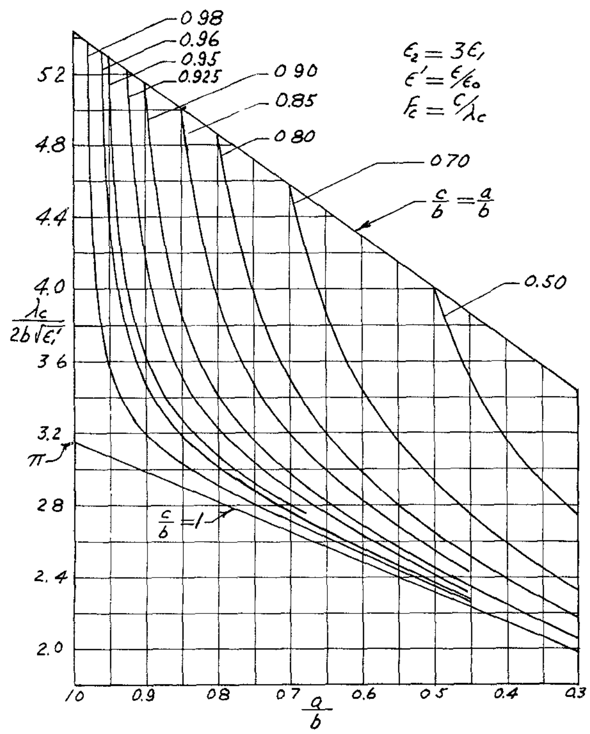

Fig. 4

In Figs. (3) (4) and (5) if inner cylinder is air, $\epsilon_1' = 1$, whereas in Fig. 6 if outer cylinder is air, $\epsilon_2' = 1$.

The phase velocities in such a structure for dominant mode waves can be determined from the variational form given by Marcuvitz.[1] The values of λ_c for a single

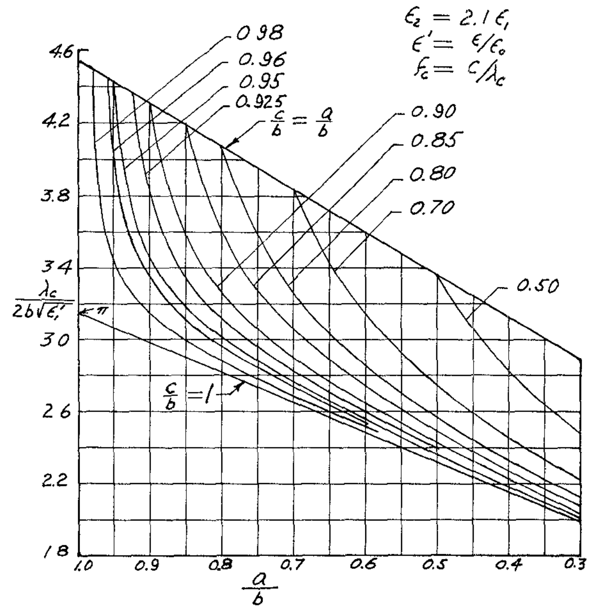

Fig. 5

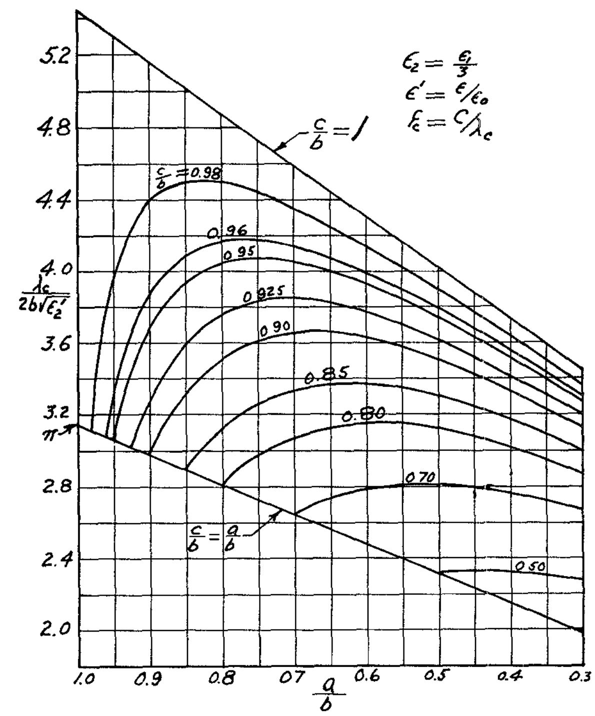

Fig. 6

homogeneous isotropic medium coaxial line (first few modes) are plotted by Moreno.[2]

Conclusion

It was found experimentally that over the band of operation only the TE_{11} resonance in air-filled choke input section was excited to a sufficient degree to be detected from input impedance measurements. In several chokes it comes in at roughly eight to twelve per cent higher frequency than that calculated from the

[1] Marcuvitz, "Waveguide Handbook," Rad. Lab. Series, vol. 10, pp. 396.

[2] Moreno, "Microwave Transmission Design," McGraw-Hill Book Co., Inc., New York, N. Y. p. 71.

long-line theoretical considerations. Presumably the difference is due to the discontinuity reactances at the ends of the choke section. However, upon inserting a dielectric cylinder partially filling the annular section, the measured shift of the resonant frequency is as predicted from the theoretical data to within the accuracy of measurement. The effective finite Q of the resonance and the degree of excitation puts limits on estimation of the center frequency of resonance. Agreement to within two to three per cent was obtained. This included mechanical tolerances, accuracy of estimated dielectric constant, and reading of the graphical data, as well as the possible error of measurement of the peak resonant frequency. From the graphical data it is quite apparent that if accurate control of the resonant TE_{11} frequency is required, one should not design such choke sections with the cylinder of higher dielectric constant very nearly filling the annular gap. The problem of machine tolerances then becomes acute.

Appendix

It may be in order to point out here that in the double dielectric system noted in Fig. 2 the only cases where we have (generally) TE_{nm} or TM_{nm} field components in such a transmission line is at the cutoff, or transverse resonant, conditions. All transmission modes above the dominant mode, except the TE_{om} and TM_{om} modes, are of the hybrid type. A generalized treatment of the coaxial line of double concentric dielectric medium[3] was called to my attention by the reviewer. The general characteristic equation given there is

$$n^2 \frac{T^2}{Q} - (F_1 - F_2)(G_1 - G_2) = 0$$

where

$$F_i = \frac{1}{V_i}\left[\frac{J_n'(V_i)N'(C_iV_i) - J_n'(C_iV_i)N_n'(V_i)}{J_n(V_i)N_n'(C_iV_i) - J_n'(C_iV_i)N_n(V_i)}\right]$$

$$G_i = \frac{\epsilon_i}{V_i}\left[\frac{J_n'(V_i)N_n(C_iV_i) - J_n(C_iV_i)N_n'(V_i)}{J_n(V_i)N_n(C_iV_i) - J_n(C_iV_i)N_n(V_i)}\right]$$

$$T^2 = \left(\frac{1}{V_1^2}\right) - \left(\frac{1}{V_2^2}\right)^2$$

$$Q = \left(\frac{\lambda_g}{\lambda}\right)^2.$$

[3] R. E. Beam and D. A. Dobson, Proceedings of The National Electronics Conference, p. 301; 1952.

In terms of the notation used in Fig. 2, Beam and Dobson's relations become

$$V_1^2 = C^2(\omega^2\mu_0\epsilon_1 - \beta_g^2)$$

$$V_2^2 = C^2(\omega^2\mu_0\epsilon_2 - \beta_g^2)$$

$$C_1 = a/c$$

$$C_2 = b/c$$

$$\text{at cutoff } V_1 = V_2\sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

$$c/\lambda_c = \frac{V_1}{2\pi\sqrt{\epsilon_1}} = \frac{V_2}{2\pi\sqrt{\epsilon_2}}$$

$$V_1 = \frac{2\pi c\sqrt{\epsilon_1}}{\lambda_c} = \frac{2\pi c}{\lambda_{1c}} = ck_{1c};$$

$$V_2 = \frac{2\pi c}{\lambda_{2c}} = ck_{2c}.$$

λ_{1c} and λ_{2c} are cutoff wavelengths in the respective medium referred to by the numerical subscript.

Using the notation in Fig. 2, the factor $F_1 - F_2$ in the first equation is

$$\frac{J_n'(ck_{1c})N_n'(ak_{1c}) - J_n'(ak_{1c})N_n'(ck_{1c})}{J_n(ck_{1c})N_n'(ak_{1c}) - J_n'(ak_{1c})N_n(ck_{1c})}\sqrt{\frac{\epsilon_2}{\epsilon_1}} - \left[\frac{J_n'\left(c\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right)N_n'\left(b\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right) - J_n'\left(b\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right)N_n'\left(c\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right)}{J_n\left(c\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right)N_n'\left(b\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right) - J_n'\left(b\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right)N_n\left(c\sqrt{\frac{\epsilon_2}{\epsilon_1}}k_{1c}\right)}\right],$$

which is the equivalent of (4) of the derivation for cutoff conditions.

Examining the first equation of the Appendix, note that as cutoff is approached the first term tends to zero. Setting one of the factors of the remaining term equal to zero is necessary to obtain a solution for the cutoff condition.

Apparently what we have done by demanding only TE_{nm} field configurations at cutoff is to determine the choice of the factor which is set equal to zero to give a solution. Choice of the magnetic Hertz vector $\overline{H} = \nabla \times \pi_z$, it appears, would indicate setting $G_1 - G_2 = 0$ for TM_{nm} cutoff conditions. Any propagation condition indicates that neither factor can be set equal to zero except for TE_{om} or TM_{om} waves (cases where the first term in the general characteristic equation is identically zero). Thus there results only hybrid modes of propagation except for $n = 0$.

Acknowledgment

I wish to thank Mr. Sanders for his active interest and aid in obtaining numerical solutions. He obtained a form of (4) which is more amenable to calculation to the required decimal place accuracy with the table of functions available. Miss Gwendolyn Brown made the numerical calculations, and plotted the curves.